THE UNIVERSITY OF TEXAS AT AUSTIN

EM 397, CSE 397: Nonlinear Static and Dynamic Finite Element Analysis (with Particular Emphasis on Solids, and an Introduction to Isogeometric Analysis)

Fall 2022

COURSE DESCRIPTION: This version of the course will be very different than previous versions in that there will be focus on the basics of isogeometric analysis. If you are expecting a traditional nonlinear finite element analysis course, you will be disappointed because this will *not* be it. So please be aware and do not register for this course unless you are interested in learning the basic technologies used in isogeometric analysis.

We will start with the geometry tools necessary to do *both* finite element and isogeometric analysis. In fact, classical finite element analysis will be derived as a special case of the more general framework of isogeometric analysis. The course will cover topics from computational geometry that comprise the foundational technologies of isogeometric analysis, namely, Bézier curves, de Casteljau algorithm, B-splines, Bernstein vs. Lagrange basis functions, degree elevation, Non-uniform Rational B-splines (NURBS), T-splines, knot insertion, Bézier elements, control polygon, variation diminishing and convex hull properties, *h*-, *p*- and *k*-refinement, trim and immersion, finite cell method, Bézier extraction, isogeometric and finite element data structures. Formulation of basic linear and nonlinear boundary-value problems and boundary/initial-value problems utilizing isogeometric and finite element basis functions.

The course and office hours will be face-to-face exclusively.

UNIQUE NUMBERS: 15294 (EM397), 65549 (CSE397)

INSTRUCTOR: Tom Hughes, hughes@oden.utexas.edu

Office hours: Tuesday and Thursday, 3:30 pm – 5:30 pm (after class, perhaps in my office in POB, perhaps at a convenient meeting place).

TAs: 1) Frimpong Baidoo, fabaidoo@utexas.edu
2) Michael Johnson, mikejohnson@utexas.edu

(Please communicate with the TAs by email to schedule Zoom office hour appointments.)

CLASS SCHEDULE: Tuesdays and Thursdays, 2:00 pm – 3:30 pm, RLP 0.120

WEB PAGE: https://canvas.utexas.edu

LEVEL: This is an advanced course aimed at graduate students in engineering, computer science,

mathematics, and the physical sciences interested in developing new knowledge, insights and skills that they may utilize in their research.

TOPICAL OUTLINE (tentative and not necessarily in this order)

01 - Introduction
02 - Bernstein Polynomials and Bezier Curves
03 - Composite Bezier Curves
04 - B-splines
05 - Computing B-spline Basis Functions
06 - Multi-dimensional B-splines
07 - Rational B-splines
08 - Computing NURBS Basis Functions
09 - Refinement of B-splines and NURBS
10 - Bezier Extraction
11 - Multi-Patch Geometries
12 - Commentary on Notation
13 - NURBS as a Basis for Analysis
14 - Boundary Value Problems - The Heat Equation
15 - Boundary Value Problems - Galerkin's Method
16 - Boundary Value Problems - Assembling the Matrix System
17 - Boundary Value Problems - Constru...ement Stiffness Matrix and Forcing Vector
18 - Boundary Value Problems - Putting It All Together
19 - Linear Elasticity - The Strong and Weak Forms
20 - Linear Elasticity - Voigt Notation
21 - Linear Elasticity - Isotropic Bodies, Plane Strain, and Plane Stress
22 - Linear Elasticity - Galerkin's Method
23 - Linear Elasticity - Assembling the Matrix System
24 - Linear Elasticity - Constructing the Element Stiffness Matrix and Forcing Vector
25 - Linear Elasticity - Putting It All Together
26 - Linear Elasticity - Alternative Constructions of the ID Array
27 - Structural Vibrations and Wave Propagation
28 - Elastodynamics - Semi-Discrete Methods and Matrix Formulation
29 - Elastodynamics - Viscous Damping
30 - Elastodynamics - From Semi-Discrete to Fully Discrete
31 - Elastodynamics - Implementation
32 - From Linear to Nonlinear - Dynamics of Hyperelastic Bodies
33 - From Linear to Nonlinear - The Generalized-alpha Method

PREREQUISITES:

There are no formal prerequisites, but the course will require some mathematical maturity, including an acquaintance with functional analysis, and familiarity with ordinary and partial differential equations, and a basic knowledge of finite element analysis.

TEXT:

Notes will be provided.

RECOMMENDED BACKGROUND READING:

Hughes, T.J.R. (2000): *The Finite Element Method – Linear Static and Dynamic Finite Element Analysis*, Dover Publications, Mineola, New York.

Cottrell, J.A., Hughes, T.J.R. and Bazilevs, Y. (2009): *Isogeometric Analysis: Toward Integration of CAD and FEA,* Wiley, Chichester, England.

Piegl, L. and Tiller, W. (1997): *The NURBS Book (Monographs in Visual Communication),* Springer-Verlag, Berlin, Heidelberg.

HOMEWORK POLICY:

There will be frequent assignments. Homework will be due at the end of the class on the due date. The homework will be graded.

TESTING AND EXAMINATION PLAN AND POLICIES:

There will be no tests or exams.

GRADING POLICY:

Grades will be based on the homework.

CLASS FORMAT:

Lectures.

ATTENDANCE:

Attendance at lectures is expected.

EVALUATION:

The course and instructor will be evaluated at the end of the semester using the approved form.

COMPUTER:

Homework assignments will involve computing.

Matlab will be suggested for many of the assignments. You may buy the student version of **Matlab** to use on your own computer at the Campus Computer Store or you can obtain it from the MathWorks website: http://www.mathworks.com

Nutils will also be used for some assignments (e.g. the finite cell method).
http://www.nutils.org/en/latest/

From the link:

Nutils is a Free and Open Source Python programming library for Finite Element Method computations, developed by Evalf Computing and distributed under the permissive MIT license. Key features are a readable, math centric syntax, an object-oriented design, strict separation of topology and geometry, and high-level function manipulations with support for automatic differentiation.

Nutils provides the tools to construct a typical simulation workflow in just a few lines of Python code, while at the same time leaving full flexibility to build novel workflows or interact with third party tools. With native support for Isogeometric Analysis (IGA), the Finite Cell method (FCM), multi-physics, mixed methods, and hierarchical refinement, **Nutils** is at the forefront of numerical discretization science. Efficient under-the-hood vectorization and built-in parallelization provide for an effortless transition from academic research projects to full-scale, real-world applications.

I may use some other software too if it provides some routines that are especially important in IGA. By the time of the course, that will be determined.

CLASS WEBSITES AND STUDENT PRIVACY:

Web-based, password-protected class sites are associated with all academic courses taught at The University. Syllabi, handouts, assignments and other resources are types of information that may be available within these sites. Site activities could include exchanging e-mail, engaging in class discussions and chats, and exchanging files. In addition, electronic class rosters will be a component of the sites. Students who do not want their names included in these electronic class rosters must

restrict their directory information in the Office of the Registrar, Main Building, Room 1. For information on restricting directory information see:

http://registrar.utexas.edu/students/records/restrictmyinfo

DROPPING COURSES:

September 10. Last day a student may drop a course for a possible refund. Last day a graduate student may, with required approvals, add a class.

October 28. Last day a graduate student may change registration in a class to or from the pass/fail basis.

December 6. Last day a graduate student may, with the required approvals, drop a class or withdraw from the University.

Further information for graduate students concerning different kinds of drops and deadlines may be found at

https://gradschool.utexas.edu/academics/policies/adding-and-dropping-courses

SPECIAL NOTES:

The University of Texas at Austin provides upon request appropriate academic adjustments for qualified students with disabilities. For more information, contact the Office of the Dean of Students at 512-471-5017, or Services for Students with Disabilities 512-471-6259.